Lowest Approximation of Relativistic Nucleon-Nucleon Scattering in Functional Quantum Theory

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Using a selfcoupled spinorfield (Fermicoupling) as a model to describe relativistic nucleon-nucleon scattering it is shown that the functional S-matrix construction for relativistic clusters in nonlinear spinortheory proposed by Stumpf reproduces in lowest order perturbation theory the result obtained by usual quantum field theoretical methods.

1. Introduction

In order to obtain theoretical information from nonlinear spinor theory[1] Stumpf and coworkers have developped a functional quantum theory[2] which enables one to calculate not only bounded states[3] but also S-matrix elements of a nonlinear quantized field, where generally the particles are given by relativistic clusters created by the field[4–9] To achieve this Stumpf introduced an universal scalarproduct in functional space[7–9]. Using that product, Englert[10] succeeded in orthonormalizing generalized free field functionals while Schäfer[11] examined the normalization of Boson functionals in nonlinear spinor theory. Calculations for the simplest scattering processes in nonlinear spinor theory are in preparation.

In this paper the above mentioned scalarproduct will be applied to a special case of a nonlinear field theory in the interaction representation, namely to the case of a selfcoupled spinorfield[1,12–15]. In the interaction representation the particles occuring are always pointparticles. They can be considered to be the simplest clusters that occur, so that for these particles Stumpf's functional S-matrix construction should reproduce the usual results, obtained by conventional methods. This will be shown in lowest order perturbation theory for a model of relativistic nucleon-nucleon scattering characterized by:

1) the mass of the nucleons is incorporated into the field equation from the very beginning,
2) the asymptotic free states satisfy the free Dirac equation with mass m,
3) the field operators satisfy canonical commutation rules,
4) the interaction is assumed to be a Fermi coupling[12–15],
5) for the S-matrix element only the lowest order of perturbation theory (contact graph) is considered. Therefore problems arising from divergencies aren't of interest in connection with this paper.

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In Section 2 usual quantum field theoretical methods are applied to calculate the S-matrix element of relativistic nucleon-nucleon scattering in lowest order perturbation theory.

In Section 3 we give the functional formulation of the relativistic nucleon-nucleon scattering. Hereby each state $|a\rangle$ of the fieldtheoretical Hilbertspace is mapped into the corresponding functional state $|\mathfrak{T}_a(j)\rangle$ with the help o the set of timeordered τ-functions belonging to $|a\rangle$. To calculate the S-matrix element one has to construct the advanced and retarded functional scattering states. Performing this, one notices that it is convenient — if not necessary — to introduce besides the timeordered τ-functions (in this paper denoted by τ^+) used by Stumpf and coworkers a new set, namely the set of antitimeordered τ-functions. For these scattering states we obtain integral-equations which are solved by iteration, leading to Neumann series.

Before evaluating these series in lowest approximation, we introduce in Section 4 the socalled physical scalarproduct in functional space which is defined via a sequence of generalized functions. To each state $|\mathfrak{T}_a(j)\rangle$ in functional space a corresponding reduced functional ε-state $|\mathfrak{T}_{a,r}(j)\rangle^\varepsilon$ may be defined, where ε defines the sequence.

Using this scalar product we show in Section 5 that the asymptotic free particles states may be orthonormalized.

In the Sections 6 and 7 we then construct the reduced functional advanced and retarded ε-scattering state in lowest approximation.

In the last section the S-matrix element of relativistic nucleon-nucleon scattering is calculated in lowest approximation. Comparison with the result obtained in Section 2 shows that the functional S-matrix construction proposed by Stumpf reproduces the result obtained by usual quantum field theoretical methods. In general spinor indices are suppressed whereas natural units ($\hbar = c = 1$) are used everywhere. Furtheron we also use nearly everywhere the generalized Einstein convention

$$f_\alpha(x)\, g_\alpha(x) := \sum_\alpha \int \mathrm{d}x\, f_\alpha(x)\, g_\alpha(x)\,.$$

Supplements are given in the appendices.

2. Quantumfieldtheoretical Formulation

The model to describe relativistic nucleon-nucleon scattering may be given here by a self coupled spinor field $\psi(x)$ with $x = (x_0, x_1, x_2, x_3)$ and may be characterized by the following Hamiltonian [12]:

$$H(t) = H(0) = \int \mathrm{d}^3\mathfrak{x}\, \overline{\psi}(x)\,(-i\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + m)\,\psi(x) + g_0 \int \mathrm{d}^3\mathfrak{x}\, :\overline{\psi}(x)\, v_\mu\, \psi(x)\, \overline{\psi}(x)\, v^\mu\, \psi(x):\, = H_0(t) + H_1(t) \quad (2.1)$$

with

γ^μ	$(\mu = 0, \ldots, 3)$	Dirac matrices,
g_0		coupling const,
$\overline{\psi} = \gamma^0 \psi^+$		adjoint field op. (+ denotes hermitean conj.),
v		one of the five lorentzinvariant couplings.

The dots mean normal ordering.

For ψ and $\overline{\psi}$ we assume the following equal time anticommutation relations:

$$[\psi(x), \psi(x')]_{+/x_0 = x_0'} = [\overline{\psi}(x), \overline{\psi}(x')]_{+/x_0 = x_0'} = 0\,, \qquad [\psi(x), \overline{\psi}(x')]_{+/x_0 = x_0'} = \gamma^0\, \delta(\mathfrak{x} - \mathfrak{x}')\,. \quad (2.2)$$

From (2.1) and (2.2) we get with Heisenberg's equation of motion the following field equations:

$$\begin{aligned} (-i\,\gamma^\mu\,\partial_\mu + m)\,\psi(x) &= -2\,g_0 :v_\mu\,\psi(x)\,\overline{\psi}(x)\,v^\mu\,\psi(x):\,, \\ (i\,\gamma^\mu\,\partial_\mu + m)\,\overline{\psi}(x) &= 2\,g_0 :v_\mu\,\overline{\psi}(x)\,\overline{\psi}(x)\,v^\mu\,\psi(x):\,. \end{aligned} \quad (2.3)$$

Now let $K = \{K^0, \mathfrak{K}; s \ldots$ energy, momentum, spin $\ldots\}$ be a complete set of quantum numbers of one nucleon, $|K_2 K_1\,\mathrm{in}\rangle$ and $|K_2' K_1'\,\mathrm{out}\rangle$ the in- and outgoing scattering state for a two nucleon system respectively, then the S-matrix element $S(K_1' K_2'; K_1 K_2)$ is given by [12, 16]:

$$S(K_1' K_2'; K_1 K_2) := \langle K_1' K_2'\,\mathrm{out} \,|\, K_2 K_1\,\mathrm{in}\rangle\,. \quad (2.4)$$

Using reduction technics one obtains [12, 16]:

$$\begin{aligned} S(K_1' K_2'; K_1 K_2) &= \tfrac{1}{2} \sum_{\lambda_1 \lambda_2} (-1)^P\, \delta(\mathfrak{K}_1' - \mathfrak{K}_{\lambda_1})\, \delta(\mathfrak{K}_2' - \mathfrak{K}_{\lambda_2}) \\ &+ \tfrac{1}{2}\,(-i/\sqrt{Z_2})^4 \int \mathrm{d}x_1 \ldots \mathrm{d}x_4\, \bar{U}(x_1 \,|\, K_1')\, \bar{U}(x_2 \,|\, K_2')\, (\overrightarrow{i\,\gamma^\mu\,\partial_\mu - m})_{x_1}\, (\overrightarrow{i\,\gamma^\mu\,\partial_\mu - m})_{x_2} \\ &\times \langle 0 |\, T\,\psi(x_1)\,\psi(x_2)\,\overline{\psi}(x_3)\,\psi(x_4)\,|\,0\rangle\, (\overleftarrow{-i\,\gamma^\mu\,\partial_\mu - m})_{x_3}\, (\overleftarrow{-i\,\gamma^\mu\,\partial_\mu - m})_{x_4}\, U(x_3 \,|\, K_2)\, U(x_4 \,|\, K_1) \end{aligned} \quad (2.5)$$

where Z_2 is the renormalization constant and

$$U(x \,|\, K) = [1/(2\pi)^{3/2}]\, \sqrt{m/K^0}\, e^{-\mathrm{i}Kx}\, u(\mathfrak{K}, s)$$

denotes the Dirac spinor $u(\mathfrak{K}, s)$ in configuration space. Using [12, 16]

$$\langle 0 |\, T\,\psi(x_1)\,\psi(x_2)\,\overline{\psi}(x_3)\,\overline{\psi}(x_4)\,|\,0\rangle = \frac{(0 |\, T\,\psi^{\mathrm{in}}(x_1) \ldots \overline{\psi}^{\mathrm{in}}(x_4)\, U(\infty, -\infty)\,|\,0)}{(0 |\, U(\infty, -\infty)\,|\,0)} \quad (2.6)$$

with

$$U(\infty, -\infty) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \mathrm{d}y_1 \ldots y_n\, T[\mathscr{H}_1(\psi^{\mathrm{in}}(y_1), \overline{\psi}^{\mathrm{in}}(y_1)) \ldots \mathscr{H}_1(\psi^{\mathrm{in}}(y_n), \overline{\psi}^{\mathrm{in}}(y_n))] \quad (2.7)$$

where $\mathscr{H}_1$ is the Hamiltonian density belonging to $H_1(t)$ then one obtains in lowest order perturbation theory

$$S(K_1' K_2'; K_1 K_2) = \tfrac{1}{2} \sum_{\lambda_1 \lambda_2} (-1)^P \delta(\mathfrak{K}_1' - \mathfrak{K}_{\lambda_1}) \delta(\mathfrak{K}_2' - \mathfrak{K}_{\lambda_2}) \tag{2.8}$$

$$- \frac{1}{Z_2^2} \frac{i g_0 m^2}{\sqrt{K_1^0 K_2^0 K_1^{0'} K_2^{0'}}} \frac{(2\pi)^4}{(2\pi)^6} \delta(K_1' + K_2' - K_1 - K_2) \sum_{\lambda_1 \lambda_2} (-1)^P \bar{u}(\mathfrak{K}_1') v_\mu u(\mathfrak{K}_{\lambda_1}) \bar{u}(\mathfrak{K}_2') v^\mu u(\mathfrak{K}_{\lambda_2}) .$$

3. Functional Formulation

First of all we put $\psi =: \psi_1$ and $\bar{\psi} =: \psi_2$ as in [6]. Then Eqs. (1.2) and (1.3) may be written in the following way:

$$[\psi_\alpha(x), \psi_{\alpha'}(x')]_{+|x_0 = x_{0'}} = i A_{\alpha\alpha'} \delta(\mathfrak{x} - \mathfrak{x}') , \tag{3.1}$$

$$(-i \Gamma_{\alpha\beta}^\mu \partial_\mu + m \delta_{\alpha\beta}) \psi_\beta(x) = g_0 V_{\alpha\beta\gamma\delta} : \psi_\beta(x) \psi_\gamma(x) \psi_\delta(x) : \tag{3.2}$$

with

$$A_{\alpha\alpha'} = \gamma^0 \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \quad \Gamma_{\alpha\beta}^\mu = \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad V_{\alpha\beta\gamma\delta} = 2 B_{\alpha\beta,\mu} C_{\gamma\delta}^{\ \mu}, \quad B_{\alpha\beta,\mu} = v_\mu \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C_{\gamma\delta}^{\ \mu} = v^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{3.3}$$

To solve such a problem one has to construct an explicit representation of the field operator satisfying (3.1), (3.2) and certain subsidiary conditions resulting from the underlying symmetry groups. This is a very difficult task which has been solved only for very simple and physically unrealistic models. To avoid these difficulties we consider single states and characterize them by their projections on a cyclic basis. In former papers of Stumpf and coworkers only projections on one cyclic basis were considered, namely the τ^+-functions defined by

$$\tau^+ \left(\begin{matrix} x_1 \dots x_n \\ a_1 \dots a_n \end{matrix} \middle| a \right) = \langle 0 | \, T \psi_{\alpha_1}(x_1) \dots \psi_{\alpha_n}(x_n) \, | a \rangle . \tag{3.4}$$

Besides these functions we use in this paper also the τ^--functions defined by

$$\tau^- \left(\begin{matrix} x_1 \dots x_n \\ a_1 \dots a_n \end{matrix} \middle| a \right) = \langle 0 | \, \bar{T} \psi_{\alpha_1}(x_1) \dots \psi_{\alpha_n}(x_n) \, | a \rangle \tag{3.5}$$

where T and $\bar{T}$ means time- and anti-time-ordering respectively. The reason is the following: The functional S-matrix-construction given by Stumpf [7-8] is based on the fieldtheoretic S-matrix-construction [17,18] by the retarded and advanced scattering state in the Schrödinger picture at the time $t = 0$. This is a very general construction as it makes no use of the possibility of splitting H into H_0 and H_1 which is necessary for the interaction representation. In order to construct the retarded and advanced scattering states one has to investigate the boundary conditions very thoroughly. Performing this one notices that it is convenient to use for the description of causal (retarded) states the τ^+-functions and for acausal (advanced) states the τ^--functions.

A similar principle is emploied by Nishijima [19]. He uses the reduction formula for retarded and advanced products for the definition of the retarded and advanced scattering states respectively. Furtheron τ^+- as well as τ^--functions are also used in Bethe-Salpeter theory [20], where τ^+ corresponds to the usual B-S-amplitude, while τ^- corresponds to the adjoint B-S-amplitude.

For these τ^+-functions a coupled system of linear integral equations may be derived by using (3.1) and (3.2). Additional these $\tau^\pm$-functions have to satisfy certain subsidiary conditions [21-23] resulting from the corresponding symmetry groups. A very elegant way to write down these equations is the functional formalism.

For the functional treatment we introduce anticommuting sources $j_\alpha(x)$ and corresponding functional derivatives $\partial_\alpha(x)$ with:

$$[j_\alpha(x), j_{\alpha'}(x')]_+ = [\partial_\alpha(x), \partial_{\alpha'}(x')]_+ = 0 , \qquad [\partial_\alpha(x), j_{\alpha'}(x')]_+ = \delta_{\alpha\alpha'}(x - x') . \tag{3.6}$$

If $x' = \Lambda x + a$ is a Poincaré transformation we assume the following transformation properties:

$$\begin{aligned} V\,\partial_\alpha(x)\,V^{-1} &= D_{\alpha\beta}\,\partial_\beta(x') \\ V\,j_\alpha(x)\,V^{-1} &= D^{-1}_{\alpha\beta}\,j_\beta(x') \end{aligned} \qquad \text{with} \quad D_{\alpha\beta} = \begin{pmatrix} S(\Lambda^{-1}) & 0 \\ 0 & S^*(\Lambda^{-1}) \end{pmatrix} \tag{3.7}$$

where S is a representation of the Poincaré group in the usual spinor space while V is one in the functional Hilbert space.

Furthermore we assume the existence of a functional ground state $|\varphi_0\rangle$ with

$$V\,|\varphi_0\rangle = |\varphi_0\rangle; \qquad \partial_\alpha(x)\,|\varphi_0\rangle = 0 \tag{3.8}$$

and the validity of the relation:

$$j_\alpha{}^+(x) = \Gamma_{\alpha\beta}{}^0\,\partial_\beta(x)\,, \qquad \partial_\alpha{}^+(x) = \Gamma_{\alpha\beta}{}^0\,j_\beta(x)\,. \tag{3.9}$$

A representation satisfying (3.6) to (3.9) was constructed by Stumpf[24]. With (3.6) and (3.8) we can interpret $j_\alpha(x)$ as a functional creation and $\partial_\alpha(x)$ as a functional destruction operator. By successive application of $j_\alpha(x)$ to $|\varphi_0\rangle$ one gets a basis in the functional space, namely the powerfunctionals[2]:

$$|D_n\left(\begin{smallmatrix} x_1 \dots x_n \\ \alpha_1 \dots \alpha_n \end{smallmatrix}\right)\rangle := (1/n!)\,j_{\alpha_1}(x_1) \dots j_{\alpha_n}(x_n)\,|\varphi_0\rangle\,. \tag{3.10}$$

Defining the adjoint power functional by

$$\overline{\langle D_n\left(\begin{smallmatrix} x_n \dots x_1 \\ \alpha_n \dots \alpha_1 \end{smallmatrix}\right)|} := (1/n!)\,\langle\varphi_0|\,\partial_{\alpha_n}(x_n) \dots \partial_{\alpha_1}(x_1) \tag{3.11}$$

and considering the scalar product divided by $\langle\varphi_0\,|\,\varphi_0\rangle$ one gets the orthonormality of the powerfunctionals:

$$\overline{\langle D_m\left(\begin{smallmatrix} x_m \dots x_1 \\ \alpha_m \dots \alpha_1 \end{smallmatrix}\right)|}\,D_n\left(\begin{smallmatrix} y_1 \dots y_n \\ \beta_1 \dots \beta_n \end{smallmatrix}\right)\rangle = \frac{1}{[n!]^2}\,\delta_{nm} \sum_{\lambda_1 \dots \lambda_n} (-1)^P\,\delta_{\alpha_1\beta_{\lambda_1}}(x_1 - y_{\lambda_1}) \dots \delta_{\alpha_n\beta_{\lambda_n}}(x_n - y_{\lambda_n})\,. \tag{3.12}$$

In order to describe the system of coupled integral equations for the $\tau^\pm$-functions we define the Generating Functional:

$$\mathfrak{T}_a{}^\pm(j) := \sum_{n=0}^{\infty} (i^n/n!)\,\tau_n{}^\pm\left(\begin{smallmatrix} x_1 \dots x_n \\ \lambda_1 \dots \alpha_n \end{smallmatrix}\,|\,a\right) j_{\alpha_1}(x_1) \dots j_{\alpha_n}(x_n)\,. \tag{3.13}$$

In full analogy to[24] the following functional equation for the functional state vector $|\mathfrak{T}_a{}^\pm(j)\rangle := \mathfrak{T}_a{}^\pm(j)\,|\varphi_0\rangle$ may be derived:

$$\begin{aligned} (-i\,\Gamma_{\alpha\beta}{}^\mu\,\partial_\mu + m\,\delta_{\alpha\beta})\,\partial_\beta(x)\,|\mathfrak{T}^\pm(j)\rangle = g_0\,V_{\alpha\beta\gamma\delta}\,\{&\partial_\beta(x)\,\partial_\gamma(x)\,\partial_\delta(x) - F_{\beta\gamma}{}^\pm(0)\,\partial_\delta(x) \\ &+ F_{\beta\delta}{}^\pm(0)\,\partial_\gamma(x) - F_{\gamma\delta}{}^\pm(0)\,\partial_\beta(x)\}\,|\mathfrak{T}^\pm(j)\rangle \mp \Gamma^0{}_{\alpha\alpha'}\,A_{\alpha'\beta}\,j_\beta(x)\,|\mathfrak{T}^\pm(j)\rangle \end{aligned} \tag{3.14}$$

where $F^\pm$ denotes the causal and acausal Feynman propagator respectively, which is defined by

$$F_{\alpha\beta}{}^+(x\,y) = \langle 0|\,T\,\psi_\alpha(x)\,\psi_\beta(y)\,|0\rangle\,, \qquad F_{\alpha\beta}{}^-(x\,y) = \langle 0|\,\bar{T}\,\psi_\alpha(x)\,\psi_\beta(y)\,|0\rangle\,. \tag{3.15}$$

The occurence of the $F^\pm(0)$ terms is equivalent to the normalordering in the field Equation (3.2). Functional differentiation of (3.14) followed by putting all sources j equal zero yields the system of coupled integral equations for the $\tau^\pm$-functions. We will not write it down as we are mainly interested in solving (3.14) by functional methods only. To do this it is convenient to consider instead of the $\mathfrak{T}$-functional the Φ-functional defined by:

$$|\mathfrak{T}^\pm(j)\rangle = \exp\{-\tfrac{1}{2}\,j_\alpha(x)\,{}^{\mathrm{f}}F_{\alpha\beta}{}^\pm(x\,y)\,j_\beta(y)\}\,|\Phi^\pm(j)\rangle \tag{3.16}$$

where ${}^{\mathrm{f}}F_{\alpha\beta}{}^\pm(x\,y)$ is the free causal and acausal Feynman propagator respectively.

For the Φ-functional Eq. (3.14) reads:

$$(-i\,\Gamma_{\alpha\beta}{}^\mu\,\partial_\mu + m\,\delta_{\alpha\beta})\,d_\beta{}^\pm(x)\,|\Phi^\pm(j)\rangle = g_0\,V_{\alpha\beta\gamma\delta}:d_\beta{}^\pm(x)\,d_\gamma{}^\pm(x)\,d_\delta{}^\pm(x):\,|\Phi^\pm(j)\rangle \mp \Gamma^0{}_{\alpha\alpha'}\,A_{\alpha\beta}\,j_\beta(x)\,|\Phi^\pm(j)\rangle \tag{3.17}$$

with $d_\alpha{}^\pm(x) = -\,{}^{\mathrm{f}}F_{\alpha\beta}{}^\pm(x\,y)\,j_\beta(y) + \partial_\alpha(x)$. The dots mean normal ordering with respect to j and ∂. This is equivalent to the occurence of the $F^\pm(0)$ terms in (3.14). In the following we are only dealing with the

free Feynman propagator. Therefore we will suppress the f. As for the free propagator $F_{\alpha\beta}{}^{\pm}(xy)$ the following equation holds:

$$(-i\,\Gamma_{\alpha\beta}{}^{\mu}\,\partial_{\mu}+m\,\delta_{\alpha\beta})\,F_{\beta\gamma}{}^{\pm}(xy)=\pm\,\Gamma_{\alpha\beta}{}^{0}\,A_{\alpha\beta}\,\delta(x-y) \tag{3.19}$$

one obtains from (3.17):

$$(-i\,\Gamma_{\alpha\beta}{}^{\mu}\,\partial_{\mu}+m\,\delta_{\alpha\beta})\,\partial_{\beta}(x)\,|\,\Phi^{\pm}(j)\rangle=g_0\,V_{\alpha\beta\gamma\delta}:d_{\beta}{}^{\pm}(x)\,d_{\gamma}{}^{\pm}(x)\,d_{\delta}{}^{\pm}(x):|\,\Phi^{\pm}(j)\rangle\,. \tag{3.19}$$

By application of the causal and acausal propagator $G_{\alpha\beta}^{(\pm)}(x-y)$ respectively, defined by

$$G_{\alpha\alpha'}^{(\pm)}(x-x')\,(-i\,\Gamma_{\alpha'\beta}^{\mu}\,\partial_{\mu}+m\,\delta_{\alpha'\beta})=\delta_{\alpha\beta}\,\delta(x-x') \tag{3.20}$$

with the same boundary conditions as $F^{\pm}$ we obtain from the causal and acausal functional states $|\,\mathfrak{T}^{\pm}(j)\rangle$ respectively the retarded and avanced functional scattering states satisfying the following equation:

$$\partial_{\alpha}(x)\,|\,\Phi^{(\pm)}(j)\rangle=g_0\,G_{\alpha\alpha'}^{(\pm)}(x-x')\;V_{\alpha'\beta\gamma\delta}:d_{\beta}{}^{\pm}(x')\,d_{\gamma}{}^{\pm}(x')\,d_{\delta}{}^{\pm}(x):|\,\Phi^{(\pm)}(j)\rangle+\partial_{\alpha}(x)\,|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle \tag{3.21}$$

where $|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle$ is a solution of the free functional equation:

$$(-i\,\Gamma_{\alpha\beta}{}^{\mu}\,\partial_{\mu}+m\,\delta_{\alpha\beta})\,\partial_{\beta}(x)\,|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle=0\,. \tag{3.22}$$

Besides (3.21) the functional states $|\,\Phi^{(\pm)}(j)\rangle$ have to satisfy certain symmetry conditions. If we restrict ourselves to the Poincaré group then these conditions are given by[21]:

$$\begin{aligned}&\mathfrak{P}_{\mu}\,|\,\Phi^{(\pm)}(j)\rangle=J_{\mu}\,|\,\Phi^{(\pm)}(j)\rangle\,,\qquad \mathfrak{P}^{2}\,|\,\Phi^{(\pm)}(j)\rangle=m^{2}\,|\,\Phi^{(\pm)}(j)\rangle\,,\\&\mathfrak{W}_{\mu}\,\mathfrak{W}^{\mu}\,|\,\Phi^{(\pm)}(j)\rangle=s(s+1)\,|\,\Phi^{(\pm)}(j)\rangle\,,\qquad \mathfrak{W}_{3}\,|\,\Phi^{(\pm)}(j)\rangle=s_3\,|\,\Phi^{(\pm)}(j)\rangle\end{aligned} \tag{3.23}$$

with the quantum numbers: $J\equiv$ 4-component total momentum $m\equiv$ mass, $s\equiv$ spin, $s_3\equiv$ spindirection and

$$\mathfrak{P}_{\mu}:=j_{\alpha}(x)\,P_{\mu}(x)\,\partial_{\alpha}(x)\,,\qquad \mathfrak{M}_{\mu\nu}:=j_{\alpha}(x)\,M_{\mu\nu}(x)\,\partial_{\alpha}(x)\,,\qquad \mathfrak{W}^{\mu}:=(1/2\,m)\,\varepsilon^{\mu\nu_1\nu_2\nu_3}\,\mathfrak{P}_{\nu_1}\,\mathfrak{M}_{\nu_2\nu_3}$$

where $P_{\mu}(x)$, $M_{\mu\nu}|\,(x)$ are representations of the infinitesimal generators of the Poincare group in ordinary spinor space.

Multiplication of (3.21) by $j_{\alpha}(x)\,P_{\lambda}(x)\,J^{\lambda}$, summation over α and integration over x yields, when combined with the first equation of (3.23) the socalled momentum averaging[25] of (3.21):

$$|\,\Phi^{(\pm)}(j)\rangle=g_0/(J_{\lambda}J^{\lambda})\,j_{\alpha}(x)\,J^{\mu}\,P_{\mu}(x)\,G_{\alpha\alpha'}^{(\pm)}(x-x')\,V_{\alpha'\beta\gamma\delta}:d_{\beta}{}^{\pm}(x')\,d_{\gamma}{}^{\pm}(x')\,d_{\delta}{}^{\pm}(x'):|\,\Phi^{(\pm)}(j)\rangle+|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle \tag{3.24}$$

or

$$(1-K^{(\pm)})\,|\,\Phi^{(\pm)}(j)\rangle=|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle \tag{3.25}$$

with

$$K^{(\pm)}=(g_0/J_{\lambda}J^{\lambda})\,j_{\alpha}(x)\,J^{\mu}\,P_{\mu}(x)\,G_{\alpha\alpha'}^{(\pm)}(x-x')\;V_{\alpha'\beta\gamma\delta}:d_{\beta}{}^{\pm}(x')\,d_{\gamma}{}^{\pm}(x')\,d_{\delta}{}^{\pm}(x'):$$

Instead of solving (3.25) exactly, which would require the knowledge of the boundary conditions for the operator $(1-K^{(\pm)})^{-1}$ we try to solve (3.25) by iteration leading directly to the Neumann series:

$$|\,\Phi^{(\pm)}(j)\rangle=\sum_{\nu=0}^{\infty}[K^{(\pm)}]^{\nu}\,|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle\,. \tag{3.26}$$

By physical reasons there do not occur any homogenious solutions of the operator $(1-K^{(\pm)})$ as the latter are just the one particle solutions according to[7], whereas here many particle solutions are constructed. For practical use it is convenient to choose the Lorentz-frame with $J=(E,0,0,0)$. With $P_0=i(\partial/\partial x_0)$ (3.26) then reads:

$$|\,\Phi^{(\pm)}(j)\rangle=\sum_{\nu=0}^{\infty}\{g_0/E\;j_{\alpha}(x)\,i\,(\partial/\partial x_0)\,G_{\alpha\alpha'}^{(\pm)}(x-x')\;V_{\alpha'\beta\gamma\delta}:d_{\beta}{}^{\pm}(x')\,d_{\gamma}{}^{\pm}(x')\,d_{\delta}{}^{\pm}(x'):\}^{\nu}\,|\overset{\text{in}}{\Phi}{}^{\text{out}}(j)\rangle\,. \tag{3.27}$$

Here one sees the full analogy to formula (4.16) of the nonrelativistic Fermion-Fermion scattering[6]. Before evaluating the Neumann series (3.27) in the lowest approximation we define the scalarproduct for physical state functionals in functional space.

4. Physical Functional Scalarproduct

The physical scalarproduct, denoted by ( ) in order to distinguish it from the scalarproduct introduced in Section 3 which was denoted by $\langle\ \rangle$, is defined via a sequence of generalized functions. Before we give the proper definition of the physical scalarproduct we will establish some preliminary definitions:

Def. 1: Let $|\mathfrak{T}_a(j)\rangle$ be a functional state

$$|\mathfrak{T}_a(j)\rangle = \sum_{n=0}^{\infty} i^n \tau_n(x_1 \ldots x_n \,|\, a)\,|\,D_n(x_1 \ldots x_n)\rangle \tag{4.1}$$

then the adjoint state $\overline{\langle\mathfrak{T}_a(j)|}$ is given by

$$\overline{\langle\mathfrak{T}_a(j)|} := \sum_{n=0}^{\infty} (-i)^n\, \overline{\tau_n(x_1 \ldots x_n \,|\, a)}\, \overline{\langle D_n(x_n \ldots x_1)|} \tag{4.2}$$

with

$$\overline{\tau_n\left(\begin{smallmatrix} x_1 \ldots x_n \\ \alpha_1 \ldots \alpha_n \end{smallmatrix} \,|\, a\right)} := \Gamma^0_{\alpha_1\beta_1} \ldots \Gamma^0_{\alpha_n\beta_n}\, \tau_n{}^*\left(\begin{smallmatrix} x_1 \ldots x_n \\ \beta_1 \ldots \beta_n \end{smallmatrix} \,|\, a\right). \tag{4.3}$$

Def. 2: Let $|\mathfrak{T}_a(j)\rangle$ be the same functional state as in Def. 1, then the functional ε-state $|\mathfrak{T}_a(j)\rangle^{\varepsilon}$ is given by replacing all Θ-functions occuring in the τ-functions by their integral representation

$$\Theta(\pm t) = \lim_{\varepsilon \to 0} \frac{\pm i}{2\pi} \int \frac{e^{i\omega t}}{\omega \pm i\,\varepsilon}\, \mathrm{d}\omega \tag{4.4}$$

without performing $\varepsilon \to 0$.

Def. 3: Let $|\mathfrak{T}_a(j)\rangle$ be the same functional state as in Def. 1. The reduced functional state $|\mathfrak{T}_{a,r}(j)\rangle$ is defined by neglecting in the τ-functions all terms with inner momentum conservation. In momentum space this is achieved if we neglect in $\tilde{\tau}(q_1 \ldots q_n \,|\, a)$ all terms in which a factor $\delta(\sum_{i=1}^{r} q_{\lambda_i})$ $(1 < r \leqq n)$ occurs.

Def. 4: From certain universal requirements[7,8] follows that the weighting operator for the physical scalarproduct should have the following form in the coordinate space:

$$\mathfrak{W}_{\varepsilon} := \sum_{n=0}^{\infty} \varepsilon^n\, g(n)\,|\,D_n(x_1 \ldots x_n)\rangle\, \overline{\langle D_n(x_n \ldots x_1)|} \tag{4.5}$$

where ε is defined in such a way that it carries the 4 dimensional scalarproduct of a free particle state over in the usual 3 dim. scalarproduct. Thereby the weighting factor $g(n)$ is equal to 1 for $n = 1$, while $g(n)$ for $n > 1$ is determined by the orthonormality of the free n-particle states.

Now we are able to give the exact definition of the physical scalarproduct:

Def. 5: Let $|\mathfrak{T}_a(j)\rangle$ and $|\mathfrak{T}_b(j)\rangle$ be two functional states, then the physical scalarproduct of these two states is given by:

$$(\mathfrak{T}_a(j)\,|\,\mathfrak{T}_b(j)) := \lim_{\varepsilon \to 0} \overline{{}^{\varepsilon}\langle\mathfrak{T}_{a,r}(j)|}\,|\mathfrak{W}_{\varepsilon}|\,\mathfrak{T}_{b,r}(j)\rangle^{\varepsilon}. \tag{4.6}$$

Using (3.12) and Appendix I we can evaluate the right hand side of (4.6) and obtain:

$$\begin{aligned}(\mathfrak{T}_a(j)\,|\,\mathfrak{T}_b(j)) &= \lim_{\varepsilon \to 0} \sum_{n=0}^{\infty} g(n)\, \varepsilon^n\, 1/[n!]^2\, \overline{\tau_n^{r,\varepsilon}(x_1 \ldots x_n \,|\, a)}\, \tau_n^{r,\varepsilon}(x_1 \ldots x_n \,|\, b) \\ &= \lim_{\varepsilon \to 0} \sum_{n=0}^{\infty} g(n)\, \varepsilon^n\, 1/\{(2\pi)^{4n}\,[n!]^2\}\, \overline{\tilde{\tau}_n^{r,\varepsilon}(q_1 \ldots q_n \,|\, a)}\, \tilde{\tau}_n^{r,\varepsilon}(q_1 \ldots q_n \,|\, b).\end{aligned} \tag{4.7}$$

Stumpf[7] and Englert[10] have established spectral representations for the τ^+-functions. For the τ^--functions the calculations run quite similar and one obtains:

$$\begin{aligned}\tilde{\tau}_n^{\pm r,\varepsilon}\left(\begin{smallmatrix} q_1 \ldots q_n \\ \alpha_1 \ldots \alpha_n \end{smallmatrix} \,|\, a\right) = \sum\!\!\!\!\!\!\int_{\mu_1 \ldots \mu_{n-1}} \Bigg(\sum_{\lambda_1 \ldots \lambda_n} (-1)^P\, M^{\pm r}\left(\begin{smallmatrix} \mu_1 \ldots \mu_{n-1},\, a \\ \alpha_{\lambda 1} \ldots \qquad \alpha_{\lambda n} \end{smallmatrix}\right) (\pm i)^{n-1}\, (2\pi)^{3n+1} \\ \times\, \delta\Big(\sum_{s=1}^{n} q_{\lambda_s} - p_a\Big) \prod_{s=1}^{n-1} \Big\{ \delta\Big(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r} - \mathfrak{p}_{\mu_s}\Big) \Big(\sum_{r=1}^{s} q^0_{\lambda_r} - p^0_{\mu_s} \pm i\,\varepsilon\Big)^{-1} \Big\} \Bigg)\end{aligned} \tag{4.8}$$

where $\sum_{\int}$ means integration or summation over a complete set of intermediate states $|\mu\rangle$. $(p_\mu{}^0, \mathfrak{p}_\mu)$ is the energy-momentum eigenvector of such a state while $p_a = (p_a{}^0, \mathfrak{p}_a)$ is that of the state $|a\rangle$. The structure function $M^{\pm r}\binom{\mu_1 \dots \mu_{n-1}, a}{\alpha_1 \dots \quad \alpha_n}$ (r denotes the reduced part) is defined by:

$$M^{\pm}\binom{\mu_1 \dots \mu_{n-1}, a}{\alpha_1 \dots \quad \alpha_n} := \langle 0 | \psi_{\alpha_1}(0) | \mu_1 \rangle \langle \mu_1 | \dots | \mu_{n-1} \rangle \langle \mu_{n-1} | \psi_{\alpha_n}(0) | a \rangle \tag{4.9}$$

where $+$ or $-$ stands whether $|a\rangle$ is a causal or an acausal state.

Inserting now (4.8) into (4.7) one gets for the physical scalarproduct of two causal and acausal functional states respectively[7,10]:

$$\begin{aligned}(\mathfrak{T}_a{}^{\pm}(j) \,|\, \mathfrak{T}_b{}^{\pm}(j)) = \lim_{\varepsilon \to 0} \sum_{n=0}^{\infty} \frac{\varepsilon^n g(n)}{n!} \Bigg\{ & \sum_{\substack{\mu_1' \dots \mu_{n-1}' \\ \mu_1 \dots \mu_{n-1}}} \int M^{\pm r}\binom{\mu_1' \dots \mu_{n-1}', a}{\alpha_1 \dots \quad \alpha_n} M^{\pm r}\binom{\mu_1 \dots \mu_{n-1}, b}{\alpha_1 \dots \quad \alpha_n} (2\pi)^{2n+2} (2\pi i)^{n-1} \\ & \times \delta(p_a - p_b) \prod_{s=1}^{n-1} \{\delta(\mathfrak{p}_{\mu_s'} - \mathfrak{p}_{\mu_s}) (\pm p_{\mu_s'}^0 \mp p_{\mu_s}^0 + 2i\varepsilon)^{-1}\} \Bigg\}\end{aligned} \tag{4.10}$$

whereas the physical scalarproduct for a causal and an acausal functional state is given by:

$$\begin{aligned}(\mathfrak{T}_a{}^{\mp}(j) \,|\, \mathfrak{T}_b{}^{\pm}(j)) = \lim_{\varepsilon \to 0} \sum_{n=0}^{\infty} \frac{\varepsilon^n g(n)}{n!} \Bigg\{ & \sum_{\substack{\mu_1' \dots \mu_{n-1}' \\ \mu_1 \dots \mu_{n-1}}} \int \overline{M^{\mp r}\binom{\mu_1' \dots \mu_{n-1}', a}{\alpha_1 \dots \quad \alpha_n}} M^{\pm r}\binom{\mu_1 \dots \mu_{n-1}, b}{\alpha_n \dots \quad \alpha_1} (2\pi)^{2n+2} (2\pi i)^{n-1} (-1)^{\binom{n}{2}} \\ & \times \delta(p_a - p_b) \prod_{s=1}^{n-1} \{\delta(\mathfrak{p}_b - \mathfrak{p}_{\mu_{n-s}} - \mathfrak{p}_{\mu_s'}) (\pm p_b{}^0 \mp p_{\mu_{n-s}}^0 \mp p_{\mu_s'}^0 + 2i\varepsilon)^{-1}\} \Bigg\}.\end{aligned} \tag{4.11}$$

The expression for $(\mathfrak{T}_a{}^{-}(j) | \mathfrak{T}_b{}^{-}(j))$ follows directly from the expression for $(\mathfrak{T}_a{}^{+}(j) | \mathfrak{T}_b{}^{+}(j))$ proved in[7,10] however formula (4.11) has to be proven. To do this we will carry over the interesting quantity $\overline{\tilde{\tau}^-} \cdot \tilde{\tau}^+$ into an expression similar to that which one obtains for $\overline{\tilde{\tau}^+} \cdot \tilde{\tau}^+$ given in[10] formula (2.17). With (4.8) we have:

$$\begin{aligned}& \overline{\tilde{\tau}_n^{-r,\varepsilon}(q_1 \dots q_n | a)}\, \tilde{\tau}_n^{+r,\varepsilon}(q_1 \dots q_n | b) \\ & = \sum_{\substack{\lambda_1 \dots \lambda_n \\ \lambda_1' \dots \lambda_n'}} (-1)^{P+P'} \Bigg\{ \sum_{\substack{\mu_1' \dots \mu_{n-1}' \\ \mu_1 \dots \mu_{n-1}}} \int \overline{M^{-r}\binom{\mu_1 \dots \mu_{n-1}, a}{\alpha_{\lambda_1} \dots \quad \alpha_{\lambda_n}}} M^{+r}\binom{\mu_1' \dots \mu_{n-1}', b}{\alpha_{\lambda_1'} \dots \quad \alpha_{\lambda_n'}} (2\pi)^{2n+2} i^{2n-2} \delta\left(\sum_{s=1}^{n} q_s - p_a\right) \prod_{s=1}^{n-1} \left\{ \delta\left(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r} - \mathfrak{p}_{\mu_s}\right) \right. \\ & \qquad \left. \times \left(\sum_{r=1}^{s} q_{\lambda_r}^0 - p_{\mu_s}^0 + i\varepsilon\right)^{-1} \right\} \delta\left(\sum_{s=1}^{n} q_s - p_b\right) \prod_{s=1}^{n-1} \left\{ \delta\left(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r'} - \mathfrak{p}_{\mu_s'}\right) \left(\sum_{r=1}^{s} q_{\lambda_r'} - p_{\mu_s'} + i\varepsilon\right)^{-1} \right\} \\ & = \sum_{\substack{\lambda_1 \dots \lambda_n \\ \lambda_1' \dots \lambda_n'}} (-1)^{P+P'} \Bigg\{ \sum_{\substack{\mu_1 \dots \mu_{n-1} \\ \mu_1' \dots \mu_{n-1}'}} \int \overline{M^{-r}\binom{\mu_1 \dots \mu_{n-1}, a}{\alpha_{\lambda_1} \dots \quad \alpha_{\lambda_n}}} M^{+r}\binom{\mu_1' \dots \mu_{n-1}', b}{\alpha_{\lambda_1'} \dots \quad \alpha_{\lambda_n'}} (2\pi)^{2n+2} (-1)^{2n-2} \\ & \qquad \times \delta\left(\sum_{s=1}^{n} q_s - p_a\right) \delta\left(\sum_{s=1}^{n} q_s - p_b\right) \prod_{s=0}^{n-1} \left\{ \delta\left(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r} - \mathfrak{p}_{\mu_s}\right) \delta\left(\sum_{r=s+1}^{n} \mathfrak{q}_{\lambda_r'} + \mathfrak{p}_{\mu_s'} - \mathfrak{p}_b\right) \right. \\ & \qquad \left. \left. \times \left(\sum_{r=1}^{s} q_{\lambda_r}^0 - p_{\mu_s}^0 + i\varepsilon\right)^{-1} \left(\sum_{r=s+1}^{n} q_{\lambda_r'}^0 - p_b^0 + p_{\mu_s'}^0 - i\varepsilon\right)^{-1} \right\} \right\}\end{aligned} \tag{4.12}$$

$$\begin{aligned}& = \sum_{\substack{\lambda_1 \dots \lambda_n \\ \lambda_1' \dots \lambda_n'}} (-1)^{P+P'+\binom{n}{2}} \Bigg\{ \sum_{\substack{\mu_1 \dots \mu_{n-1} \\ \mu_1' \dots \mu_{n-1}'}} \int \overline{M^{-r}\binom{\mu_1 \dots \mu_{n-1}, a}{\alpha_{\lambda_1} \dots \quad \alpha_{\lambda_n}}} M^{+r}\binom{\mu_1' \dots \mu_{n-1}', b}{\alpha_{\lambda_n'} \dots \quad \alpha_{\lambda_1'}} (2\pi)^{2n+2} \\ & \qquad \times \delta\left(\sum_{s=1}^{n} q_s - p_a\right) \delta\left(\sum_{s=1}^{n} q_s - p_b\right) \prod_{s=1}^{n-1} \left\{ \delta\left(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r} - \mathfrak{p}_{\mu_s}\right) \delta\left(\sum_{r=1}^{s} \mathfrak{q}_{\lambda_r'} + \mathfrak{p}_{\mu_{n-s}'} - \mathfrak{p}_b\right) \right. \\ & \qquad \left. \left. \times \left(\sum_{r=1}^{s} q_{\lambda_r'}^0 - p_b^0 + p_{\mu_{n-s}'}^0 - i\varepsilon\right)^{-1} \left(\sum_{r=1}^{s} q_{\lambda_r}^0 - p_{\mu_s}^0 + i\varepsilon\right)^{-1} \right\} \right\}.\end{aligned}$$

Replacing now in (4.12) $\mathfrak{p}_b - \mathfrak{p}_{\mu' n-s}$ by $\mathfrak{p}_{\mu' s}$ and $p_b^0 - p_{\mu' n-s}^0$ by $p_{\mu' s}^0$ we essentially obtain formula (2.17) of[10]. Therefore it is sufficient, to perform essentially the inverse replacements in (4.10) in order to obtain (4.11) taking care of the interchange $\mu_i \to \mu'_i$.

5. Asymptotic Free Functional States

Free functional states are characterized by satisfying (3.22) and (3.23). First of all we consider one particle states. Let $K = \{K^0, \mathfrak{K}, s, m, \ldots\}$ denote a complete set of quantum numbers, then the in- as well as the outgoing free functional one particle states $|\overset{\text{in}}{\Phi}{}^{\text{out}}(j, K)\rangle$ are given by[24]

$$|\overset{\text{in}}{\Phi}{}^{\text{out}}(j, K)\rangle = i f_\beta(x|K)\, j_\beta(x)\, |\varphi_0\rangle \tag{5.1}$$

where (cf. Appendix II)

$$f_\beta(x|K) = \langle 0|\, \psi_\beta(x)\, |K\rangle = \begin{cases} \dfrac{1}{(2\pi)^{3/2}}\, u(\mathfrak{K}, s)\, e^{iKx} & \text{for} \quad \beta = 1 \\ 0 & \text{for} \quad \beta = 2 \end{cases} \tag{5.2}$$

denotes a one particle solution of the free Dirac equation

$$(-i\, \Gamma_{\alpha\beta}{}^\mu\, \partial_\mu + m\, \delta_{\alpha\beta})\, f_\beta(x|K) = 0\,. \tag{5.3}$$

As for the S-matrix construction we only need the scalarproduct of advanced and retarded states we restrict ourselves on the calculation of the physical scalarproduct of out- and ingoing free functional states. For the one particle states we obtain with (3.16) and the translational invariance of $F^\pm$:

$$\begin{aligned}(\mathfrak{T}^{\text{out}}(j, K')\,|\,\mathfrak{T}^{\text{in}}(j, K)) &= (\Phi^{\text{out}}(j, K')\,|\,\Phi^{\text{in}}(j, K)) \\ &= \lim_{\varepsilon\to 0} \varepsilon\, g(1)\, \bar{f}_\beta(x|K')\, f_\beta(x|K) = \lim_{\varepsilon\to 0} \varepsilon\, 2\pi\, \delta(0)\, \delta(\mathfrak{K} - \mathfrak{K}')\,.\end{aligned} \tag{5.4}$$

This undefined expression may be transformed into a welldefined one, if we define the ε-function $f_\beta{}^\varepsilon(x|K)$ in an obvious way. As

$$f_\beta(x|K) = f_\beta(x|K)\,[\Theta(t) + \Theta(-t)] = f_\beta(x|K) \lim_{\varepsilon\to 0} \frac{i}{2\pi} \int e^{-i\omega t} \left[\frac{1}{\omega + i\,\varepsilon} - \frac{1}{\omega - i\,\varepsilon}\right] \mathrm{d}\omega \tag{5.5}$$

we define $f_\beta{}^\varepsilon(x|K)$ by:

$$f_\beta{}^\varepsilon(x|K) := f_\beta(x|K)\, \frac{i}{2\pi} \int e^{-i\omega t} \left[\frac{1}{\omega + i\,\varepsilon} - \frac{1}{\omega - i\,\varepsilon}\right] \mathrm{d}\omega\,. \tag{5.6}$$

Inserting this in (5.4) then residual integration yields:

$$(\mathfrak{T}^{\text{out}}(j, K')\,|\,\mathfrak{T}^{\text{in}}(j, K)) = \lim_{\varepsilon\to 0} \varepsilon \bar{f}_\beta{}^\varepsilon(x|K')\, f_\beta{}^\varepsilon(x|K) = \delta(\mathfrak{K} - \mathfrak{K}')\,. \tag{5.7}$$

Comparing (5.7) and (5.4) one obtains the formal identity: $\lim_{\varepsilon\to 0} \varepsilon\, 2\pi\, \delta(0) := 1\,.$ (5.8)

Defining in (5.1) the one particle creation operator by: $\mathfrak{A}^+(K) := i f_\beta(x|K)\, j_\beta(x)$ (5.9)

then the free functional n-particle states are given by:

$$\begin{aligned}|\overset{\text{in}}{\Phi}{}^{\text{out}}(j, K_1 \ldots K_n)\rangle &= \frac{1}{\sqrt{n!}}\, \mathfrak{A}^+(K_1) \ldots \mathfrak{A}^+(K_n)\, |\varphi_0\rangle = (i^n/\sqrt{n!})\, f(x_1|K_1) \ldots f(x_n|K_n)\, j(x_1) \ldots j(x_n)\, |\varphi_0\rangle \\ &= (i^n/n!)\, 1/\sqrt{n!}\, \{\sum_{\lambda_1 \ldots \lambda_n} (-1)^P f(x_1|K_{\lambda_1}) \ldots f(x_n|K_{\lambda_n})\}\, j(x_1) \ldots j(x_n)\, |\varphi_0\rangle \\ &=: (i^n/n!)\, \varphi_n{}^\pm(x_1 \ldots x_n|K_1 \ldots K_n)\, j(x_1) \ldots j(x_n)\, |\varphi_0\rangle\,.\end{aligned} \tag{5.10}$$

The corresponding $\mathfrak{T}$-functional reads:

$$|\overset{\text{in}}{\mathfrak{T}}{}^{\text{out}}(j, K_1 \dots K_n)\rangle = \exp\{-\tfrac{1}{2} j(x)\, F^{\pm}(xy)\, j(y)\}\, |\overset{\text{in}}{\Phi}{}^{\text{out}}(j, K_1 \dots K_n)\rangle$$
$$= \sum_{\mu=0}^{\infty} \frac{i^{n+2\mu}}{(n+2\mu)!}\, \tau^{\pm}{}_{n+2\mu}(x_1 \dots x_{n+2\mu} \,|\, K_1 \dots K_n)\, j(x_1) \dots j(x_{n+2\mu})\, |\, \varphi_0\rangle \tag{5.11}$$

where the $\tau^{\pm}$-functions are given by a specialization of formula (II.7) of[26]:

$$\tau^{\pm}{}_{n+2\mu}(x_1 \dots x_{n+2\mu} \,|\, K_1 \dots K_n) = \sum_{\lambda_1 \dots \lambda_{n+2\mu}} (-1)^P \frac{(-1)^{\mu}}{2^{\mu}\, \mu!\, n!}\, \varphi n^{\pm}(x_{\lambda_1} \dots x_{\lambda_n} K_1 \dots K_n)\, F^{\pm}(x_{\lambda_{n+1}} x_{\lambda_{n+2}}) \dots$$
$$F^{\pm}(x_{\lambda_{n+2\mu-1}} x_{\lambda_{n+2\mu}}). \tag{5.12}$$

In momentum space this formula reads:

$$\tilde{\tau}_{n+2\mu}{}^{\pm}(q_1 \dots q_{n+2\mu} \,|\, K_1 \dots K_n) = \sum_{\lambda_1 \dots \lambda_{n+2\mu}} (-1^P) \frac{(-1)^{\mu}}{2^{\mu}\, \mu!\, n!}\, \tilde{\varphi} n^{\pm}(q_{\lambda_1} \dots q_{\lambda_n} \,|\, K_1 \dots K_n)$$
$$\times\, \tilde{F}^{\pm}(q_{\lambda_{n+1}} q_{\lambda_{n-2}}) \dots \tilde{F}^{\pm}(q_{\lambda_{n+2\mu-1}} q_{\lambda_{n+2\mu}}). \tag{5.13}$$

From $F^{\pm}(xy) = F^{\pm}(x-y)$ it follows that $\tilde{F}^{\pm}(q_1 q_2) = \delta(q_1 + q_2)\, \tilde{F}^{\pm}(q_1)$. Therefore according to Def. 3 the reduced $\tau^{\pm}$-functions are given by

$$\tilde{\tau}_{\varrho}{}^{\pm r}(q_1 \dots q_{\varrho} \,|\, K_1 \dots K_n) = \delta_{\varrho n}\, \tilde{\varphi} n^{\pm}(q_1 \dots q_n \,|\, K_1 \dots K_n) = \delta_{\varrho n}\, \tilde{\tau} n^{\pm}(q_1 \dots q_n \,|\, K_1 \dots K_n). \tag{5.14}$$

From here one obtains for the reduced ε-$\tilde{\tau}^{\pm}$-functions:

$$\tilde{\tau}_{\varrho}^{\pm r,\varepsilon}(q_1 \dots q_{\varrho} \,|\, K_1 \dots K_n) = \delta_{\varrho n}\, \tilde{\tau} n^{\pm,\varepsilon}(q_1 \dots q_n \,|\, K_1 \dots K_n) = \delta_{\varrho n}\, \tilde{\varphi}^{\pm}{}_{n,\varepsilon}(q_1 \dots q_n \,|\, K_1 \dots K_n) \tag{5.15}$$

with this relation the scalarproduct of a free functional outgoing m-particle state and a free functional ingoing n-particle state reads with (4.7):

$$(\mathfrak{T}^{\text{out}}(j, K_1' \dots K_m') \,|\, \mathfrak{T}^{\text{in}}(j, K_1 \dots K_n))$$
$$= \delta_{nm} \frac{1}{[n!]^2}\, g(n) \lim_{\varepsilon \to 0} \overline{\tilde{\tau} n^{-\varepsilon}(q_1 \dots q_n \,|\, K_1 \dots K_n)}\, \tilde{\tau} n^{+\varepsilon}(q_1 \dots q_n \,|\, K_1 \dots K_n)\, \varepsilon^n (2\pi)^{-4n}. \tag{5.16}$$

A straight foreward calculation similar to that of[10] gives the following spectral representation for the $\tilde{\tau} n^{\pm}$-functions (spinor indices σ_i are written down explicitly):

$$\tilde{\tau} n^{\pm,\varepsilon}\left(\begin{matrix} q_1 \dots q_n \\ 1\sigma_1 \dots 1\sigma_n \end{matrix} \,\middle|\, K_1 \dots K_n\right) = \frac{(\pm i)^{n-1}}{\sqrt{n!}}\, (2\pi)^{3n+1} \sum_{\lambda_1 \dots \lambda_n} (-1)^P \int \mathrm{d}p_1 \dots \mathrm{d}p_{n-1} \tag{5.17}$$
$$\times \sum_{\nu_1 \dots \nu_n} (-1)^P \prod_{l=1}^{n} f_{\sigma_{\lambda l}}(\mathfrak{K}_{\nu_l})\, \delta\left(\sum_{j=1}^{n} (q_j - K_j)\right) \prod_{j=1}^{n-1} \left\{ \delta\left(p_j - \sum_{s=1}^{j} K_{\nu_s}\right) \delta\left(\sum_{s=1}^{j} \mathfrak{q}_{\lambda_s} - \mathfrak{p}_j\right) \left(\sum_{s=1}^{j} q_{\lambda_s}^0 - p_j^0 \pm i\varepsilon\right)^{-1} \right\}.$$

By comparison with (4.8) one directly obtains the structure function for a free functional n-particle state:

$$M^r\left(\begin{matrix} p_1 \dots p_{n-1}, \sum_{i=1}^{n} K_i \\ 1\sigma_1 \dots \quad 1\sigma_n \end{matrix}\right) = 1/\sqrt{n!} \sum_{\nu_1 \dots \nu_n} (-1)^P \prod_{l=1}^{n} f_{\sigma_l}(\mathfrak{K}_{\nu_l}) \prod_{j=1}^{n-1} \delta\left(p_j - \sum_{s=1}^{j} K_{\nu_s}\right). \tag{5.18}$$

Noting (5.18) and (4.11) we get for the scalarproduct (5.16) after some straight forward calculations:

$$(\mathfrak{T}^{\text{out}}(j, K_1' \dots K_m') \,|\, \mathfrak{T}^{\text{in}}(j, K_1 \dots K_n))$$
$$= \delta_{nm} \lim_{\varepsilon \to 0} \frac{g(n)}{n!} \int \mathrm{d}p_1 \dots \mathrm{d}p_{n-1}\, \mathrm{d}p_1' \dots \mathrm{d}p'_{n-1}\, \overline{M^r\left(\begin{matrix} p'_1 \dots p'_{n-1}, \sum_{i=1}^{n} K_i' \\ 1\sigma_1 \dots \quad 1\sigma_n \end{matrix}\right)}$$
$$\times M^r\left(\begin{matrix} p_1 \dots p_{n-1}, \sum_{i=1}^{n} K_i \\ 1\sigma_n \dots \quad 1\sigma_1 \end{matrix}\right) (-1)\, \tbinom{n}{2}\, (2\pi)^{2n+2}\, (2\pi i)^{n-1}\, \delta\left(\sum_{i=1}^{n} (K_i - K_i')\right)$$
$$\times \prod_{s=1}^{n-1} \left\{ \delta\left(\sum_{i=1}^{n} \mathfrak{K}_i - \mathfrak{p}_{n-s} - \mathfrak{p}_s'\right) \left(\sum_{i=1}^{n} K_i^0 - p_s^{0\prime} - p^0{}_{n-s} + 2i\varepsilon\right)^{-1} \right\} \tag{5.19}$$
$$= \delta_{nm} \frac{1}{n!} \sum_{\nu_1 \dots \nu_n} (-1)^P \prod_{j=1}^{n} \delta(\mathfrak{K}_j' - \mathfrak{K}_{\nu j})$$

when we use (5.8) and choose

$$g(n) := 2^{n-1}\,. \tag{5.20}$$

Thus we have obtained the orthonormality of the free functional out- and ingoing states.

6. Two Particle Scattering Functional in Lowest Approximation

Let K_1K_2 and $K_1'K_2'$ be the two complete sets of the two in- and outgoing particles respectively. According to (5.10) the corresponding in- and outgoing free functional state is given by:

$$|\overset{\text{in}}{\Phi^{\text{out}}}(j, {}^{K_1K_2}_{K_1'K_2'})\rangle = \frac{i^2}{2}\,\varphi_2\left({}^{x_1\,x_2}_{\alpha_1\,\alpha_2}\,\middle|\,{}^{K_1K_2}_{K_1'K_2'}\right) j_{\alpha_1}(x_1)\, j_{\alpha_2}(x_2)\,|\varphi_0\rangle\,. \tag{6.1}$$

Our aim is to evaluate the Neumann series (3.27) in the lowest approximation. First of all we perform the normalordering in $K^{\pm}$. This yields:

$$\begin{aligned} K^{\pm} = \frac{g_0}{E}\, j_\alpha(x)\,\frac{i\,\partial}{\partial t}\, G^{(\pm)}_{\alpha\alpha'}(x-x')\,\{&- V_{\alpha'\beta\gamma\delta}\,F^{\pm}_{\beta\beta'}(x'-y')\,F^{\pm}_{\gamma\gamma'}(x'-z')\,F^{\pm}_{\delta\delta'}(x'-u') \\ &\times j_{\beta'}(y')\,j_{\gamma'}(z')\,j_{\delta'}(u') + W_{\alpha'\beta\gamma\delta}\,F^{\pm}_{\beta\beta'}(x'-y')\,F^{\pm}_{\gamma\gamma'}(x'-z')\,j_{\beta'}(y')\,j_{\gamma'}(z')\,\partial_\delta(x') \\ &- \hat{V}_{\alpha'\beta\gamma\delta}\,F^{\pm}_{\beta\beta'}(x'-y')\,j_{\beta'}(y')\,\partial_\gamma(x')\,\partial_\delta(x') + V_{\alpha'\beta\gamma\delta}\,\partial_\beta(x')\,\delta_\gamma(x')\,\partial_\delta(x')\}\end{aligned} \tag{6.2}$$

with

$$W_{\alpha\beta\gamma\delta} := V_{\alpha\beta\gamma\delta} - V_{\alpha\beta\delta\gamma} - V_{\alpha\delta\gamma\beta}\,, \qquad \hat{V}_{\alpha\beta\gamma\delta} := V_{\alpha\beta\gamma\delta} - V_{\alpha\gamma\beta\delta} - V_{\alpha\delta\gamma\beta}\,. \tag{6.3}$$

From (3.27) we have for $\nu = 1$:

$$|\Phi^{(\pm)}(j, K_1K_2)\rangle = K^{\pm}\,|\overset{\text{in}}{\Phi^{\text{out}}}(j, K_1K_2)\rangle + |\overset{\text{in}}{\Phi^{\text{out}}}(j, K_1K_2)\rangle =: |\hat{\Phi}^{(\pm)}(j, K_1K_2)\rangle + |\overset{\text{in}}{\Phi^{\text{out}}}(j, K_1K_2)\rangle. \tag{6.4}$$

Combining (6.1) and (6.2) we get after some elementary calculations:

$$\begin{aligned} &|\hat{\Phi}^{(\pm)}(j, K_1K_2)\rangle \\ &\quad= \frac{g_0}{E}\,i\,\frac{\partial}{\partial t_1}\,G^{(\pm)}_{\alpha_1\alpha}(x_1-x)\,V_{\alpha\beta\gamma\delta}\,F^{\pm}_{\beta\alpha_2}(x-x_2)\,F^{\pm}_{\gamma\alpha_3}(x-x_3)\,F^{\pm}_{\delta\alpha_4}(x-x_4)\,\frac{1}{\sqrt{2}}\,f_{\alpha_5}(x_5\,|\,K_1) \\ &\qquad\times f_{\alpha 6}(x_6\,|\,K_2)\,j_{\alpha 1}(x_1)\ldots j_{\alpha 6}(x_6)\,|\varphi_0\rangle \\ &\quad+ \frac{g_0}{E}\,i\,\frac{\partial}{\partial t_1}\,G^{(\pm)}_{\alpha_1\alpha}(x_1-x)\,W_{\alpha\beta\gamma\delta}\,F^{\pm}_{\beta\alpha_2}(x-x_2)\,F^{\pm}_{\gamma\alpha_3}(x-x_3)\,\frac{1}{\sqrt{2}}\left\{\sum_{\lambda_1\lambda_2}(-1)^P f_{\alpha_4}(x_4|\,K_{\lambda_1})\,f_\delta(x\,|\,K_{\lambda_2})\right\} \\ &\qquad\times j_{\alpha_1}(x_1)\ldots j_{\alpha_4}(x_4)\,|\varphi_0\rangle \qquad (6.5) \\ &- \frac{g_0}{E}\,i\,\frac{\partial}{\partial t_1}\,G^{(\pm)}_{\alpha_1\alpha}(x_1-x)\,\hat{V}_{\alpha\beta\gamma\delta}\,F^{\pm}_{\beta\alpha_2}(x-x_2)\,\frac{1}{\sqrt{2}}\left\{\sum_{\lambda_1\lambda_2}(-1)^P f_\gamma(x\,|\,K_{\lambda_1})\,f_\delta(x\,|\,K_{\lambda_2})\right\} j_{\alpha_1}(x_1)\,j_{\alpha_2}(x_2)\,|\varphi_0\rangle. \end{aligned}$$

As our starting point for defining the physical scalarproduct was the functional state $|\mathfrak{T}_a(j)\rangle$ given in (4.1) with totally antisymmetric τ-functions we have to transform (6.5) into the standard form given by:

$$|\Phi_a(j)\rangle = \sum_{n=0}^{\infty}\frac{i^n}{n!}\,\varphi_n(x_1\ldots x_n\,|\,a)\,j(x_1)\ldots j(x_n)\,|\varphi_0\rangle \tag{6.6}$$

with the totally antisymmetric φ-functions:

$$\varphi_n(x_1\ldots x_n\,|\,a) = \frac{1}{i^n}\,\partial(x_n)\ldots\partial(x_1)\,|\Phi(j,a)\rangle|_{j=0} \tag{6.7}$$

Using (6.5) and (6.7) we get for the $\hat{\varphi}^{(\pm)}$-functions of $|\hat{\Phi}^{(\pm)}(j, K_1K_2)\rangle$:

$$\begin{aligned} \hat{\varphi}_2{}^{(\pm)}\left({}^{x_1x_2}_{\alpha_1\alpha_2}\,\middle|\,K_1K_2\right) = \frac{g_0}{E}\sum_{\mu_1\mu_2}(-1)^P\left\{i\,\frac{\partial}{\partial t_{\mu_1}}\,G^{(\pm)}_{\alpha_{\mu 1}\alpha}(x_{\mu_1}-x)\,\hat{V}_{\alpha\beta\gamma\delta}\,F^{\pm}_{\beta\alpha_{\mu 2}}(x-x_{\mu_2})\right\} \\ \times\frac{1}{\sqrt{2}}\left\{\sum_{\lambda_1\lambda_2}(-1)^{P'} f_\gamma(x\,|\,K_{\lambda_1})\,f_\delta(x\,|\,K_{\lambda_2})\right\}\end{aligned} \tag{6.8}$$

and similar expressions for $\hat{\varphi}_4^{(\pm)}$ and $\hat{\varphi}_6^{(\pm)}$ which are of no interest in connection with this paper, as we are only interested in first order terms of the coupling constant g_0 in the S-matrix element.

As the functional states $|\,\mathfrak{T}^{(\pm)}(j, K_1K_2)\rangle$ and $|\,\Phi^{(\pm)}(j, K_1K_2)\rangle$ are combined by the transformation (3.16) which is equal to that of free functional states (5.11) we have as in (5.14)·

$$|\,\mathfrak{T}_r^{(\pm)}(j, K_1K_2)\rangle = |\,\Phi_r^{(\pm)}(j, K_1K_2)\rangle\,. \tag{6.9}$$

In order to get the reduced $\Phi^{\pm}$-functional state we consider the fouriertransform of (6.8) which reads (cf. App. I and III):

$$\tilde{\hat{\varphi}}_2^{(\pm)}\left(\begin{smallmatrix}q_1\,q_2\\ \alpha_1\alpha_2\end{smallmatrix}\middle|\,K_1K_2\right) = \frac{g_0}{E}(2\pi)^4\,\delta(q_1+q_2-K_1-K_2)\,\frac{1}{\sqrt{2}}\left\{\sum_{\lambda_1\lambda_2}(-1)^P f_\gamma(\mathfrak{K}_{\lambda_1})\,f_\delta(\mathfrak{K}_{\lambda_2})\right\}$$
$$\times\,\hat{V}_{\alpha\beta\gamma\delta}\left\{\sum_{\mu_1\mu_2}(-1)^P\sqrt{m^2+\mathfrak{q}^2{}_{\mu_1}}\,\tilde{G}^{(\pm)}_{\alpha_{\mu 1}\alpha}(q_{\mu_1})\,\tilde{F}^{(\pm)}_{\beta\alpha_{\mu 2}}(-q_{\mu_2})\right\}. \tag{6.10}$$

According to Def. 3 of Section 4 the reduced functional scattering state then is given by

$$|\,\mathfrak{T}_r^{(\pm)}(\tilde{j}, K_1K_2)\rangle = |\,\Phi_r^{(\pm)}(\tilde{j}, K_1K_2)\rangle = |\,\hat{\Phi}_r^{(\pm)}(\tilde{j}, K_1K_2)\rangle + |\overset{\text{in}}{\Phi}{}^{\text{out}}(\tilde{j}, K_1K_2)\rangle$$
$$= (i^2/2)\,\tilde{\varphi}_2^{(\pm)}(q_1q_2\,|\,K_1K_2)\,\tilde{j}(q_1)\,\tilde{j}(q_2)\,|\,\varphi_0\rangle \tag{6.11}$$

with
$$\tilde{\varphi}_2^{(\pm)}(q_1q_2\,|\,K_1K_2) := \tilde{\hat{\varphi}}_2^{(\pm)}(q_1q_2\,|\,K_1K_2) + \tilde{\varphi}_2(q_1q_2\,|\,K_1K_2)\,.$$

Before closing this section we will examine the $\tilde{\hat{\varphi}}_2^{(\pm)}$-function in some more detail.

According to (5.2) we have $f_2 \equiv 0$. Therefore only the quantities $\hat{V}_{1111}$, $\hat{V}_{1211}$, $\hat{V}_{2111}$ and $\hat{V}_{2211}$ are of interest. From (3.3) and (6.3) we obtain:

$$\hat{V}_{1111} = \hat{V}_{2111} = \hat{V}_{2211} = 0\,, \qquad \hat{V}_{1211} = 2\,v_\mu \otimes v^\mu\,. \tag{6.12}$$

Using the fact that $\tilde{F}_{11}{}^{\pm} = \tilde{F}_{22}{}^{\pm} = \tilde{G}_{12}{}^{(\pm)} = \tilde{G}_{21}{}^{(\pm)} = 0$ and $\tilde{F}_{\alpha\beta}(q) = -\tilde{F}_{\beta\alpha}(-q)$ then one obtains with (6.12) and $\tilde{G}_{11}{}^{(\pm)} =: \tilde{G}^{(\pm)}$, $\tilde{F}_{12}{}^{\pm} =: \tilde{F}^{\pm}$ from (6.10):

$$\tilde{\hat{\varphi}}_2^{(\pm)}\left(\begin{smallmatrix}q_1q_2\\ 1\,1\end{smallmatrix}\middle|\,K_1K_2\right) = -\,(\sqrt{2}\,g_0/E)\,\delta(K_1+K_2-q_1-q_2)\left\{\sum_{\lambda_1\lambda_2}(-1)^P\,(\sqrt{m^2+\mathfrak{q}_1{}^2}\,\tilde{G}^{(\pm)}(q_1)\,v_\mu\right.$$
$$\left.\times\,f(\mathfrak{K}_{\lambda_1})\,\tilde{F}^{\pm}(q_2)\,v^\mu f(\mathfrak{K}_{\lambda_2}) - \sqrt{m^2+\mathfrak{q}_2{}^2}\,\tilde{F}^{\pm}(q_1)\,v^\mu f(\mathfrak{K}_{\lambda_2})\,\tilde{G}^{(\pm)}\,v_\mu f(\mathfrak{K}_{\lambda_1})\right\} \tag{6.13}$$

while the other $\hat{\varphi}_2{}^{(\pm)}$-functions ($\equiv \hat{\varphi}_2{}^{(\pm)}$-functions with at least one $\alpha_i = 2$, $i = 1,2$) are zero.

7. Spectral Representation

The aim of this section is the determination of the functional retarded and advanced ε-scattering state in lowest approximation.

First of all we consider the spectral representation of a general $\tilde{\tau}_2{}^{\pm}$-function with respect to a two particle state $|\,K\rangle$. It reads according to (4.8):

$$\tilde{\tau}_2{}^{\pm}\left(\begin{smallmatrix}q_1q_2\\ \alpha_1\alpha_2\end{smallmatrix}\middle|\,K\right) = \int \mathrm{d}p\left\{\sum_{\lambda_1\lambda_2}(-1)^P M^{\pm r}\left(\begin{smallmatrix}p,K\\ \alpha\lambda_1\,\alpha\lambda_2\end{smallmatrix}\right)(2\pi)^7\,(\pm i)\,\delta(q_1+q_2-K)\,\delta(\mathfrak{q}_{\lambda_1}-\mathfrak{p})\,(q^0_{\lambda_1}-p^0\pm i\,\varepsilon)^{-1}\right\}. \tag{7.1}$$

Now we will prove the following

statement: The spectral representation of the quantity $\tilde{\hat{\varphi}}_2{}^{(\pm)}$ as given in (6.13) is that of a general $\tilde{\tau}_2{}^{\pm}$-function but with an ε dependend structure function. Putting $\hat{\omega} = \sqrt{\mathfrak{p}^2+m^2}$ and $K := K_1+K_2$ the latter is given by:

$$M_{\varepsilon}{}^{(\pm)r}\,{}^{(p,\,K}_{1\sigma_1\,1\sigma_2)} = \delta(p^0-\hat{\omega})\,\frac{g_0\sqrt{2}}{K^0\,p^0(2\pi)^3}\left[\frac{1}{K^0-2\,p^0\pm 2\,i\,\varepsilon}-\frac{1}{K^0}\right]$$
$$\times(p^0\gamma^0-\mathfrak{p}\cdot\boldsymbol{\gamma}+m)_{\sigma_1}\,v_\mu f(\mathfrak{K}_1)\,(p^0\gamma^0+\mathfrak{p}\cdot\boldsymbol{\gamma}+m)_{\sigma_2}v^\mu f(\mathfrak{K}_2)$$
$$+\,\delta(p^0-K^0-\hat{\omega})\,\frac{g_0\sqrt{2}}{K^0(p^0-K^0)\,(2\pi)^3}\left[\frac{1}{K^0}-\frac{1}{K^0-2\,p^0\mp 2\,i\,\varepsilon}\right]$$
$$\times((p^0-K^0)\,\gamma^0-\mathfrak{p}\cdot\boldsymbol{\gamma}+m)_{\sigma_1}\,v_\mu f(\mathfrak{K}_1)\,((p^0-K^0)\,\gamma^0+\mathfrak{p}\cdot\boldsymbol{\gamma}+m)_{\sigma_2}v^\mu f(\mathfrak{K}_2)\,. \tag{7.2}$$

proof: Inserting in (6.13) the expressions for $\tilde{F}^{\pm}$ and $\tilde{G}^{(\pm)}$ from App. III, we obtain with $K:=K_1+K_2$ and $\omega:=\omega_1=\sqrt{m^2+\mathfrak{q}_1{}^2}=\omega_2=\sqrt{m^2+\mathfrak{q}_2{}^2}$ (as $\mathfrak{K}\equiv 0$)

$$\tilde{\tilde{q}}_{2,\varepsilon}^{(\pm)}\,{}^{(q_1q_2}_{1\,1}\,|\,K_1K_2) = \pm\,\frac{i\,g_0}{2\,K^0\,\omega}\,(2\pi)^4\sqrt{2}\,\delta(q_1+q_2-K)\prod_{j=1}^{2}\left\{\frac{1}{q_j{}^0-\omega\pm i\,\varepsilon}-\frac{1}{q_j{}^0+\omega\mp i\,\varepsilon}\right\}$$
$$\times\Big[\sum_{\lambda_1\lambda_2}(-1)^P(\bar{q}_1+m)\,v_\mu f(\mathfrak{K}_{\lambda_1})\,(\bar{q}_2+m)\,v^\mu f(\mathfrak{K}_{\lambda_2})\Big]\,. \tag{7.3}$$

Using now the relation

$$\prod_{j=1}^{2}\left\{\frac{1}{q_j{}^0-\omega\pm i\,\varepsilon}-\frac{1}{q_j{}^0+\omega\mp i\,\varepsilon}\right\}\delta(q_1{}^0+q_2{}^0-K^0)$$
$$=\delta(q_1{}^0+q_2{}^0-K^0)\left[\sum_{\mu=1}^{2}\left\{\left[\frac{1}{K^0-2\,\omega\pm 2\,i\,\varepsilon}-\frac{1}{K^0}\right]\frac{1}{q_\mu{}^0-\omega\pm i\,\varepsilon}\right.\right. \tag{7.4}$$
$$\left.\left.+\left[\frac{1}{K^0}-\frac{1}{K^0+2\,\omega\mp 2\,i\,\varepsilon}\right]\frac{1}{q_\mu{}^0-K^0-\omega\pm i\,\varepsilon}\right\}\right]$$

we obtain from (7.3) after some elementary calculations:

$$\tilde{\tilde{q}}_2{}^{(\pm)}\,{}^{(q_1q_2}_{1\,1}\,|\,K_1K_2) = \pm\,\frac{i\,g_0(2\pi)^4\sqrt{2}}{\omega\,K^0}\,\delta(q_1+q_2-K^0)\left[\sum_{\lambda_1\lambda_2}(-1)^P\,(\omega\,\gamma^0-\mathfrak{p}\cdot\boldsymbol{\gamma}+m)\right.$$
$$\times\,v_\mu f(\mathfrak{K}_1)\,(\omega\,\gamma^0+\mathfrak{p}\cdot\boldsymbol{\gamma}+m)\,v^\mu f(\mathfrak{K}_2)\left\{\left[\frac{1}{K^0-2\,\omega\pm 2\,i\,\varepsilon}-\frac{1}{K^0}\right]\frac{1}{q_{\lambda_1}^0-\omega\pm i\,\varepsilon}\right. \tag{7.5}$$
$$\left.\left.+\left[\frac{1}{K^0}-\frac{1}{K^0+2\,\omega\mp 2\,i\,\varepsilon}\right]\frac{1}{q-K^0-\omega\pm i\,\varepsilon}\right\}\right]$$

or, when introducing an additional integration over p and putting $\hat{\omega}:=\sqrt{m^2+\mathfrak{p}^2}$ we have

$$\tilde{\tilde{q}}_{2,\varepsilon}^{(\pm)}\,{}^{(q_1q_2}_{1\,1}\,|\,K_1K_2) = \pm\,\frac{i\,g_0\sqrt{2}\,(2\pi)^4}{K^0}\,\delta(q_1+q_2-K)\left[\sum_{\lambda_1\lambda_2}(-1)^P\,\delta(\mathfrak{q}_{\lambda_1}-\mathfrak{p})\right.$$
$$\times\left\{\delta(p^0-\hat{\omega})\,\frac{1}{p^0}\,(p^0\gamma^0-\mathfrak{p}\cdot\boldsymbol{\gamma}+m)f(\mathfrak{K}_1)\,(p^0\gamma^0+\mathfrak{p}\cdot\boldsymbol{\gamma}+m)\,v^\mu f(\mathfrak{K}_2)\left[\frac{1}{K^0-2\,p^0\pm 2\,i\,\varepsilon}-\frac{1}{K^0}\right]\right.$$
$$+\,\delta(p^0-K^0-\hat{\omega})\,\frac{1}{p^0-K^0}\,((p^0-K^0)\,\gamma^0-\mathfrak{p}\cdot\boldsymbol{\gamma}+m)\,v_\mu f(\mathfrak{K}_1)\,((p^0-K^0)\,\gamma^0+\mathfrak{p}\cdot\boldsymbol{\gamma}+m)$$
$$\left.\left.\times\,v^\mu f(\mathfrak{K}_2)\left[\frac{1}{K^0}+\frac{1}{K^0+2\,p^0\pm 2\,i\,\varepsilon}\right]\right\}\frac{1}{q-p^0\pm i\,\varepsilon}\right]. \tag{7.6}$$

Comparison with (7.1) now directly yields the structure function (7.2), q.e.d.

With (5.15), (6.11) and (7.6) we now obtain for the functional advanced and retarded reduced ε-scattering state in lowest approximation:

$$|\,\mathfrak{T}_r{}^{(\pm)}(\tilde{j},K_1K_2)\rangle^{\varepsilon} = \frac{i^2}{2!}\,\tilde{\tilde{q}}_2{}^{(\pm)}(q_1q_2\,|\,K_1K_2)\,\tilde{j}(q_1)\,\tilde{j}(q_2)\,|\,\varphi_0\rangle + \frac{i^2}{2!}\,\tilde{\varphi}_{2,\varepsilon}(q_1q_2\,|\,K_1K_2)\,\tilde{j}(q_1)\,\tilde{j}(q_2)\,|\,\varphi_0\rangle$$
$$=:|\,\hat{\mathfrak{T}}_r{}^{(\pm)}(\tilde{j},K_1K_2)\rangle^{\varepsilon}+|\,\mathfrak{T}_r{}^{\overset{\text{in}}{\text{out}}}(\tilde{j},K_1K_2)\rangle^{\varepsilon}\,. \tag{7.7}$$

8. Functional S-Matrix in Lowest Approximation

In the Heisenberg picture the S-matrix element is defined by the scalarproduct of the corresponding advanced and retarded scattering state[17,18]:

$$S_{ab} := \langle a^{(-)} \mid b^{(+)} \rangle . \tag{8.1}$$

According to Stumpf[7-9] the corresponding S-matrix element in functional space is given by

$$S_{ab} = (\mathfrak{T}^{(-)}(j,a) \mid \mathfrak{T}^{(+)}(j,b)) = \lim_{\varepsilon \to 0} {}^{\varepsilon}\langle \overline{\mathfrak{T}_r^{(-)}(j,a)} \mid \mathfrak{W}_\varepsilon \mid \mathfrak{T}_r^{(+)}(j,b) \rangle^{\varepsilon} . \tag{8.2}$$

Inserting (7.7) into (8.2), one obtains

$$\begin{aligned} S(K_1' K_2'; K_1 K_2) &= (\mathfrak{T}^{(-)}(j, K_1' K_2') \mid \mathfrak{T}^{(+)}(j, K_1 K_2)) \\ &= (\mathfrak{T}^{\text{out}}(j, K_1' K_2') \mid \mathfrak{T}^{\text{in}}(j, K_1 K_2)) + (\hat{\mathfrak{T}}^{(-)}(j, K_1' K_2') \mid \mathfrak{T}^{\text{in}}(j, K_1 K_2)) \\ &\quad + (\mathfrak{T}^{\text{out}}(j, K_1' K_2') \mid \hat{\mathfrak{T}}^{(+)}(j, K_1 K_2)) + (\hat{\mathfrak{T}}^{(-)}(j, K_1' K_2') \mid \hat{\mathfrak{T}}^{(+)}(j, K_1 K_2)) . \end{aligned} \tag{8.3}$$

As we are only interested in linear terms of the coupling constant g_0 in the S-matrix element, it is sufficient, to consider the first three terms in (8.3). The first term was just considered in Section 5, where we found the result (5.19). The calculation of the other two terms will be performed now using (4.11), (5.18) and (7.2). Taking into account the limes procedure $\varepsilon \to 0$, one notices that from (7.2) only those terms contribute which have the denominator $K^0 - 2p^0 \pm 2i\varepsilon$. Therefore we obtain with (5.20):

$$\begin{aligned} &(\mathfrak{T}^{\text{out}}(j, K_1' K_2') \mid \hat{\mathfrak{T}}^{(+)}(j, K_1 K_2)) = \lim_{\varepsilon \to 0} \frac{\varepsilon^2}{2!} g(2) \int \mathrm{d}p \, \mathrm{d}p' \, \overline{M^r \binom{p' K'}{1\sigma_1 \, 1\sigma_2}} \\ &\quad \times M_\varepsilon^{(+)r} \binom{p, K}{1\sigma_2 \, 1\sigma_1} (2\pi)^6 \, 2\pi i \, (-1) \, \delta(\mathfrak{K} - \mathfrak{p} - \mathfrak{p}') \, \frac{1}{K^0 - p^0 - p^{0\prime} + 2i\varepsilon} \, \delta(K - K') \\ &= \lim_{\varepsilon \to 0} \varepsilon^2 \int \mathrm{d}p \, \mathrm{d}p' \, \frac{1}{\sqrt{2}} \sum_{\lambda_1 \lambda_2} (-1)^P \bar{f}_{\sigma_1}(\mathfrak{K}_{\lambda_1'}) \bar{f}_{\sigma_2}(\mathfrak{K}_{\lambda_2'}) \, \delta(K_{\lambda_1'} - p') \, \delta(p^0 - \hat{\omega}) \\ &\quad \times \frac{g_0 \sqrt{2}}{K^0 p^0 (2\pi)^3} \, \frac{1}{K^0 - 2p^0 + 2i\varepsilon} \, (p^0 \gamma^0 - \mathfrak{p} \cdot \boldsymbol{\gamma} + m)_{\sigma_2} v_\mu f(\mathfrak{K}_1) \, (p^0 \gamma^0 + \mathfrak{p} \cdot \boldsymbol{\gamma} + m)_{\sigma_1} v^\mu f(\mathfrak{K}_2) \\ &\quad \times (2\pi)^6 \, 2\pi i (-1) \, \delta(\mathfrak{K} - \mathfrak{p} - \mathfrak{p}') \, (K^0 - p^0 - p^{0\prime} + 2i\varepsilon)^{-1} \, \delta(K - K') \\ &= \sum_{\lambda_1 \lambda_2} (-1)^P \bar{f}_{\sigma_1}(\mathfrak{K}'_{\lambda_1}) \bar{f}_{\sigma_2}(\mathfrak{K}'_{\lambda_2}) \, \frac{g_0 (2\pi)^4}{K^0 K_{\lambda_2}^{0\prime}} \, (-i) \, (K_{\lambda_2}^{0\prime} \gamma^0 - \mathfrak{K}'_{\lambda_2} \cdot \boldsymbol{\gamma} + m)_{\sigma_2} v_\mu f(\mathfrak{K}_1) \\ &\quad \times (K_{\lambda_1}^{0\prime} \gamma^0 - \mathfrak{K}'_{\lambda_1} \cdot \boldsymbol{\gamma} + m)_{\sigma_1} v^\mu f(\mathfrak{K}_2) \, (2i)^{-2} \, \delta(K - K') . \end{aligned} \tag{8.4}$$

Using now (A 2.8) we get:

$$\begin{aligned} (\mathfrak{T}^{\text{out}}(j, K_1' K_2') \mid \hat{\mathfrak{T}}^{(+)}(j, K_1 K_2)) &= -i(2\pi)^4 \, \delta(K - K') \, \frac{g_0 m^2}{K^0 (2\pi)^6} \\ &\quad \times \sum_{\lambda_1 \lambda_2} (-1)^P \, \frac{1}{K_{\lambda_2}^{0\prime}} \, \bar{u}(\mathfrak{K}_{\lambda_1'}) \, v_\mu f(\mathfrak{K}_1) \, \bar{u}(\mathfrak{K}_{\lambda_2'}) \, v^\mu f(\mathfrak{K}_2 . \end{aligned} \tag{8.5}$$

In the same way we obtain

$$\begin{aligned} &(\hat{\mathfrak{T}}^{(-)}(j, K_1' K_2') \mid \mathfrak{T}^{\text{in}}(j, K_1 K_2)) \\ &\quad = -i(2\pi)^4 \, \delta(K - K') \, \frac{g_0 m^2}{K^0 (2\pi)^6} \sum_{\lambda_1 \lambda_2} (-1)^P \, \frac{1}{K_{\lambda_2}^0} \, u(\mathfrak{K}_{\lambda_1}) \, \gamma^0 v_\mu^* f^*(\mathfrak{K}_1') \, u(\mathfrak{K}_{\lambda_2}) \, \gamma^0 v^{\mu *} f^*(\mathfrak{K}_{\lambda_2}) . \end{aligned} \tag{8.6}$$

Using the relation[16]

$$\gamma^0 v^* = v \gamma^0$$

valid for all five lorentzinvariant couplings and noticing that we are working in the special frame with $\mathfrak{K} \equiv 0$, which implies together with the δ-function occuring in (8.5) and (8.6) that $K_1{}^0 = K_2{}^0 = K_1{}^{0\prime} = K_2{}^{0\prime}$ we may combine (8.5) and (8.6). Thus we get for the S-matrix element in lowest approximation:

$$S(K_1' K_2'; K_1 K_2) = \frac{1}{2!} \sum_{\lambda_1 \lambda_2} (-1)^P \delta(\mathfrak{K}_1' - \mathfrak{K}_{\lambda_1}) \delta(\mathfrak{K}_2' - \mathfrak{K}_{\lambda_2}) \tag{8.7}$$

$$- i (2\pi)^4 \delta(K_1' + K_2' - K_1 - K_2) \frac{g_0 m^2}{(2\pi)^6 \sqrt{K_1{}^{0\prime} K_2{}^{0\prime} K_1{}^0 K_2{}^0}} \sum_{\lambda_1 \lambda_2} (-1)^P \bar{u}(\mathfrak{K}_1') v_\mu u(\mathfrak{K}_{\lambda_1}) \bar{u}(\mathfrak{K}_{\lambda_2}) v^\mu u(\mathfrak{K}_{\lambda_2})$$

which is just the result obtained in (2.8), when using conventional reduction technics and perturbation theory. Therefore in the lowest approximation the functional approach and the conventional approach are equivalent.

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Appendix I

Relations between coordinate space and momentum space:
Defining:

$$j_\alpha(x) = \int e^{iqx} \tilde{j}_\alpha(q)\, \mathrm{d}q\,, \qquad \partial_\alpha(x) = 1/(2\pi)^4 \int e^{-iqx} \tilde{\partial}_\alpha(q)\, \mathrm{d}q \tag{A1.1}$$

it follows from (1.2):

$$[\tilde{j}_\alpha(q), \tilde{j}_{\alpha'}(q')]_+ = [\tilde{\partial}_\alpha(q), \tilde{\partial}_{\alpha'}(q')]_+ = 0\,, \qquad [\tilde{\partial}_\alpha(q), \tilde{j}_{\alpha'}(q')]_+ = \delta_{\alpha\alpha'} \delta(q - q')\,. \tag{A1.2}$$

As $F(xy)$ and $G(xy)$ are translation invariant one has:

$$\begin{aligned} \tilde{F}(q_1 q_2) &= \int \exp\{i(q_1 x_1 + q_2 x_2)\} F(x_1 x_2)\, \mathrm{d}x_1\, \mathrm{d}x_2 = (2\pi)^4 \delta(q_1 + q_2) \tilde{F}(q_1)\,, \\ \tilde{G}(q_1 q_2) &= \int \exp\{i(q_1 x_1 + q_2 x_2)\} G(x_1 x_2)\, \mathrm{d}x_1\, \mathrm{d}x_2 = (2\pi)^4 \delta(q_1 + q_2) \tilde{G}(q_1)\,. \end{aligned} \tag{A1.3}$$

Defining now

$$\tau_n(x_1 \ldots x_n \mid a) = 1/(2\pi)^{4n} \int \exp\left\{-i \sum_{j=1}^{n} x_j q_j\right\} \tilde{\tau}_n(q_1 \ldots q_n \mid a)\, \mathrm{d}q_1 \ldots \mathrm{d}q_n \tag{A1.4}$$

then the $\mathfrak{T}$-functional in momentum space is given by

$$\mid \mathfrak{T}_a(\tilde{j}) \rangle = \sum_{n=0}^{\infty} \frac{i^n}{n!} \tilde{\tau}_n(q_1 \ldots q_n \mid a) \tilde{j}(q_1) \ldots \tilde{j}(q_n) \mid \varphi_0 \rangle\,. \tag{A1.5}$$

Appendix II

The free Dirac-field and the normalization of one particle states.
We begin with the well-known decomposition of the free Dirac field [16]:

$$\psi(\mathfrak{x}, t) = \sum_{\pm s} \int \frac{\mathrm{d}^3\mathfrak{p}}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}} \{b(\mathfrak{p}, s)\, u(\mathfrak{p}, s)\, e^{-ipx} + d^+(\mathfrak{p}, s)\, v(\mathfrak{p}. s)\, e^{ipx}\} \tag{A2.1}$$

and a similar formula for $\psi^+(\mathfrak{x}, t)$. A basis in the corresponding fieldtheoretical Hilbert space is given by successive application of the creation operators b^+ and d^+ to the vacuum $\mid 0 \rangle$. The scalarproduct of these basical states should be defined relativistically invariant, as this is true for power functionals (2.5). According to [27] this means for one-particle states $\mid K \rangle$ and $\mid K' \rangle$:

$$\langle K' \mid K \rangle = (K^0/m)\, \delta(\mathfrak{K} - \mathfrak{K}')\,. \tag{A2.2}$$

This can be achieved by introducing new creation- and destruction-operators b', b'^+, d', d'^+ with:

$$[b'(\mathfrak{p}, s), b'^+(\mathfrak{p}', s')]_+ = [d'(\mathfrak{p}, s), d'^+(\mathfrak{p}, s'')]_\mp = (p^0/m)\,\delta(\mathfrak{p} - \mathfrak{p}')\,\delta_{ss'} \tag{A2.3}$$

while the other anticommutators vanish. The connection between the primed and unprimed operators is given by:

$$a'(\mathfrak{p}, s) = \sqrt{p^0/m}\, a(\mathfrak{p}, s) \quad \text{with} \quad a \in \{b, b^+, d, d^+\} \tag{A2.4}$$

or expressed by the states:

$$|K\rangle = b^{+\prime}(\mathfrak{K}, s)\,|0\rangle = \sqrt{K^0/m}\, b^+(\mathfrak{K}, s)\,|0\rangle\,. \tag{A2.5}$$

With (A2.1) and (A2.5) we obtain for the matrixelement $\langle 0|\,\psi_\alpha(x)\,|K\rangle$ for $\alpha = 1$:

$$\langle 0|\,\psi(x)\,|K\rangle = \frac{1}{(2\pi)^{3/2}}\, u(\mathfrak{K}, s)\, e^{-iKx} =: f(\mathfrak{K})\, e^{-iKx} \tag{A2.6}$$

for $\alpha = 2$:

$$\langle 0|\,\bar{\psi}(x)\,|K\rangle = 0 \tag{A2.6}$$

where $u(\mathfrak{K}, s)$ is the usual Dirac spinor satisfying

$$(K^0\gamma^0 - \mathfrak{K}\cdot\boldsymbol{\gamma} - m)\,u(\mathfrak{K}, s) = 0 \quad \text{and} \quad \bar{u}(\mathfrak{K}, s)\,u(\mathfrak{K}, s') = \delta_{ss'}\,. \tag{A2.7}$$

Furthermore the equation $\bar{u}(\mathfrak{K}, s)\,(K^0\gamma^0 - \mathfrak{K}\cdot\boldsymbol{\gamma} - m) = 0$ implies the useful formula:

$$\bar{f}(\mathfrak{K})\,(K^0\gamma^0 - \mathfrak{K}\cdot\boldsymbol{\gamma} + m) = 2m\bar{f}(\mathfrak{K})\,. \tag{A2.8}$$

Appendix III

The free causal and acausal Feynman propagator $F^\pm_{\alpha\beta}(xy)$ and the causal and acausal propagator $G_{\alpha\beta}{}^{(\pm)}(xy)$:

Using (2.13), (2.15) and from [16] the formulas (6.47) and (13.72) we obtain with $\Lambda_+(p) = \{\pm(p^0\gamma^0 - \mathfrak{p}\cdot\boldsymbol{\gamma}) + m\}\,(2m)^{-1}$:

$$F^\pm(x_1x_2) = \int \mathrm{d}\mathfrak{p}\,(m/p^0(2\pi)^{-3})\,\{\Theta(\pm(t_1 - t_2))\,\Lambda_+(p)\exp\{-ip(x_1 - x_2)\} + \Theta(\pm(t_2 - t_1))\,\Lambda_-(p)\exp\{ip(x_1 - x_2)\}\}\,, \tag{A3.1}$$

$$G^{(\pm)}(x_1x_2) = \pm i\int \mathrm{d}\mathfrak{p}\,(m/p^0(2\pi)^{-3})\,\{\Theta(\pm(t_1 - t_2))\,\Lambda_+(p)\exp\{-ip(x_1 - x_2)\} + \Theta(\pm(t_2 - t_1))\,\Lambda_-(p)\exp\{ip(x_1 - x_2)\}\}\,. \tag{A3.2}$$

In momentum space they read:

$$\tilde{F}^\pm(q_1q_2) = \pm i(2\pi)\,\delta(q_1 + q_2)\,(q_1^0\gamma^0 - \mathfrak{q}_1\cdot\boldsymbol{\gamma} + m)/\sqrt{2\,\mathfrak{q}_1^2 + m^2} \times\left\{\frac{1}{q_1{}^0 - \sqrt{\mathfrak{q}_1^2 + m^2} \pm i\varepsilon} - \frac{1}{q_1{}^0 + \sqrt{\mathfrak{q}_1^2 + m^2} \mp i\varepsilon}\right\} \tag{A3.3}$$

$$\tilde{G}^{(\pm)}(q_1q_2) = -(2\pi)\,\delta(q_1 + q_2)\,\frac{(q_1^0\gamma^0 - \mathfrak{q}_1\cdot\boldsymbol{\gamma} + m)}{2\sqrt{\mathfrak{q}_1^2 + m^2}} \times\left\{\frac{1}{q_1{}^0 - \sqrt{\mathfrak{q}_1^2 + m^2} \pm i\varepsilon} - \frac{1}{q_1{}^0 + \sqrt{\mathfrak{q}_1^2 + m^2} \mp i\varepsilon}\right\}. \tag{A3.4}$$

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